1976-05

Analysis of deficits in discrete time resource allocation problems with correlated supplies and demands

Marshall, Kneale T.

Monterey, California. Naval Postgraduate School

NPS-55MtRh76051

NAVAL POSTGRADUATE SCHOOL

Monterey, California

ANALYSIS OF DEFICITS IN

DISCRETE TIME RESOURCE ALLOCATION PROBLEMS

WITH CORRELATED SUPPLIES AND DEMANDS

by

K. T. Marshall

and

F. R. Richards

May 1976

Approved for public release; distribution unlimited.

Prepared for:
Office of Naval Research, Arlington, Virginia 22217

NAVAL POSTGRADUATE SCHOOL
Monterey, California

Rear Admiral Linder
Superintendent

Jack R. Borsting
Provost

The work reported herein was supported by the Foundation Research Program of the Naval Postgraduate School with funds provided by the Chief of Naval Research.

Prepared by:

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NPS 55MtRh760506	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Analysis of Deficits in Discrete Time Resource Allocation Problems with Correlated Supplies and Demands		5. TYPE OF REPORT & PERIOD COVERED Technical Report 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) K. T. Marshall F. R. Richards		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61152N, RR 000-01-10 N0001476WR60052
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research Foundation Arlington, Virginia 22217		12. REPORT DATE May 1976 13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Budgeting
Queueing
Resource Allocation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper is primarily concerned with the stochastic behavior of deficits in discrete time resource allocation problems when demands for resources are random and when future allocations are based on past demands. The effects of various allocation policies are analyzed, and the sequence of deficits is shown to be related to waiting times and queue sizes in queuing systems. A number of applications are described, and a budgeting problem is used to illustrate the results.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

TABLE OF CONTENTS

Introduction

Let $\{X_i\}$ and $\{B_i\}$, $i = 0,1,2,...,$ be sequences of nonnegative random variables and define the sequence $\{D_i\}$ by

$$D_{n+1} = \max\left(0, D_n + X_n - B_n\right),\ n=0,1,2,... \qquad (1)$$

Equation (1) will be familiar to any reader who has studied the single channel queue with FIFO order of service. If X_n is the service time of customer n and B_n is the time between arrivals of customers n and n+1, then, if customer 0 starts a busy period ($D_0 = 0$), D_n represents the delay in queue of customer n (see, for example, Kleinrock [1975], p. 278)

The purpose of this paper is to analyze the distribution of D_n for a class of correlations between $\{X_i\}$ and $\{B_i\}$. In section 1 we give a number of examples, including a budgeting problem, which can be modelled by equation (1) with such correlations. This budgeting problem is used to illustrate our analyses. In section 2 we analyze the case $B_n = X_{n-1}$. In section 3 we extend our results to the case where B_n is a convex linear combination of X_{n-1} through $X_{n-\ell}$ for some $\ell \geq 1$. In section 4 we interpret our result for the G/G/1 queue, and in section 5 we suggest areas for future study.

1. Examples

Consider a budgeting process in discrete time, where demands for funds occur over time, and unused funds budgeted in a given

period cannot be carried forward, but where unfilled demands are carried forward. Examples of such budget processes occur in government agencies operating from one fiscal year to the next. If X_n is the demand for funds in period n, and B_n the budget allocated to period n, then D_{n+1} is the budget deficit at the start of period $n+1$. For an example in the U. S. Navy supply system, see Daeschner [1975]. The budgeting problem is the example that we pursue in this paper. However, the following examples also fit the same structure of equation (1). It is left to the reader to interpret our results for these examples.

Consider a job shop which is scheduled in discrete time periods. Let X_n be the demand in period n for some resource such as manhours. Such a resource, if not used in the given period, is lost. Let B_n be the amount of resource allocated by a planning process for period n. Although unused resources like manhours cannot be carried forward from period to period, unfilled demands for these resources must eventually be satisfied. Thus, D_n represents the work backlog at the start of period n.

Consider now a reservoir control problem. Let the water output in period n be B_n and assume that the input to the reservoir in period n is X_n. Let D_n be the reservoir content at the start of period n. Then the sequence $\{D_n\}$ satisfies equation (1) (assuming infinite reservoir capacity).

Consider next a periodic-review inventory process with a single perishable item. Let X_n be the demand in period n and B_n the stock which arrives in period n. Let D_n be the amount on back-order at the start of period n. Again the sequence $\{D_n\}$ satisfies (1).

Finally, equation (1) can be given the following queueing interpretation. Consider a service system which is reviewed periodically. Let X_n be the total number of arrivals in period n and B_n the total service capacity in period n. Then D_{n+1} represents the "queue" at the start of period $n+1$.

Unlike the simple GI/G/1 queue discussed in the introduction, in the above examples it may not be realistic to assume that the two sequences $\{X_i\}$ and $\{B_i\}$ are independent or that each is a sequence of iid random variables. In the following sections we investigate the behavior of D_n for a number of cases where the two sequences are correlated.

We end this section with a well known result (see, for example, Kleinrock [1975], p. 278), that if $U_n = X_n - B_n$, then

$$D_{n+1} = \text{Max}\left(0, U_n, U_n + U_{n-1}, \ldots, U_n + U_{n-1} + \ldots + U_1 + U_0\right) \quad (2)$$

Equation (2) which is equivalent to equation (1) will be important in our analyses.

2. Single Period Correlation

In this and later sections we focus on the budgeting example discussed earlier. Let the sequence of demands $\{X_i\}$ be iid random variables with finite first moment μ and distribution function F, and let the budget allocated to period n be equal to the demand in period $n-1$. Thus,

$$B_n = X_{n-1}, \qquad n \geq 1. \tag{3}$$

Let B_o be an independent random variable distributed as X_o. From (3) and the definition of U_n we have,

$$U_n + U_{n-1} + \ldots + U_{n-i} = X_n - X_{n-i-1}, \qquad i = 0,1,\ldots,n-1 \tag{4}$$

and

$$U_n + U_{n-1} + \ldots + U_0 = X_n - B_0.$$

Thus, from (2)

$$D_{n+1} = \text{Max}\left(0, X_n - X_{n-1}, X_n - X_{n-2}, \ldots, X_n - X_0, X_n - B_0\right)$$

which can be rewritten as

$$D_{n+1} = \text{Max}\left(0, X_n - \text{Min}\,(X_{n-1}, X_{n-2}, \ldots, X_0, B_0)\right). \tag{5}$$

Now, let $m = \inf\{x | F(x) > 0\}$. Clearly, m is the minimum demand that could occur in any period. If we let

$$Y_n = \text{Min}\,(X_{n-1}, X_{n-2}, \ldots, X_0, B_0),$$

then

$$D_{n+1} = \mathrm{Max}(0, X_n - Y_n) \quad \text{and} \tag{6}$$

$$P[Y_n > y] = [1-F(y)]^n = [\bar{F}(y)]^n, \tag{7}$$

where $\bar{F}(y) = 1 - F(y)$.

By using conditional probability arguments, we determine the distribution of D_{n+1} from (6) and (7) to be

$$P[D_{n+1} \le x] = F(x^-) + \int_{x-}^{\infty} \left[\bar{F}\left((u-x)^-\right)\right]^n dF(u), \qquad x \ge 0. \tag{8}$$

From the definition of m,

$$\bar{F}(x)^n \to 1 \quad \text{for } x < m \text{ and } \bar{F}(x)^n \to 0 \quad \text{for } x \ge m.$$

Thus we see that $\{D_n\}$ converges in distribution to a random variable, say D, with distribution function $D(x)$, where

$$D(x) = F(m + x), \qquad x \ge 0.$$

From (8) it is straight forward to show that

$$E(D_{n+1}) = \mu - \int_0^{\infty} \bar{F}(u)^{n+1} du,$$

and therefore,

$$E(D) = \mu - m. \tag{9}$$

The above equations show that the budgeting process can be operated without planned surplus resources, i.e. $E[B_n] = E[X_n]$,

and deficits remain small and well behaved. In the language of queues, (9) would indicate that the single channel queue can be operated with traffic intensity

$$\rho = \frac{E[B_n]}{E[X_n]} = 1$$

and with finite (indeed small!) waiting times. This result needs closer scrutiny, and we return to it in section 4.

Let i be the index of a period with $D_i = 0$ and let $K(>i)$ be the first succeeding period with zero deficit. Now let $N = K - i+1$. In queueing theory, N is analogous to the number of customers served in a busy period. Now

$$\{N>n\} \Leftrightarrow \{U_1>0, U_1 + U_2>0, \ldots, U_1 + U_2 + \ldots + U_n > 0\}. \tag{10}$$

Using (3) and (4), (10) can be shown to be

$$\{N>n\} \Leftrightarrow \{X_1>X_o, X_2>X_o, \ldots, X_n>X_o\}.$$

Since the demands $\{X_n\}$ are assumed to be iid,

$$P(N>n) = \int_0^\infty \bar{F}(u)^n dF(u) = \frac{1}{n+1}, \qquad n = 0,1,2,\ldots \tag{11}$$

independent of the demand distribution F. Thus, although the "busy periods" are finite with probability 1, they have an infinite

mean. This would lead us to believe that if S_n is the surplus resource at the end of period n (which is lost at the beginning of period $n + 1$), then, for large n, S_n would be zero with probability 1. We now show this to be the case. First, note that

$$S_n = \text{Max}\Big(0, B_n - (D_n + X_n)\Big) .$$

Combining this equation with (1) yields

$$D_{n+1} - S_n = D_n + X_n - B_n$$

where $D_{n+1}S_n = 0$. Taking expected values in this equation gives

$$E[S_n] = 0.$$

Here we have assumed n large so that $E[D_{n+1}] = E[D_n]$. Since S_n is a non-negative random variable, it must be zero with probability 1.

3. Multiperiod Correlation.

In this section we extend the results in section 2 by letting the budget in period n be a function of the demands in periods $n-1, n-2, \ldots, (n-\ell)$ for some fixed integer ℓ. Specifically, let $a_i \geq 0$ and $\sum_1^\ell a_i = 1$, and let

$$B_n = \sum_{i=1}^{\ell} a_i X_{n-i}, \qquad n=0,1,2,... \tag{12}$$

For $n < \ell$, we require the introduction of the additional random variables $X_{-1}, X_{-2}, \ldots, X_{-\ell}$ so that B_n will be well-defined. As with $\{X_i\}_{i=0}^{\infty}$ we assume that $\{X_{-1}, X_{-2}, \ldots, X_{-\ell}\}$ is a set of iid random variables with distribution function F. Note that $\{a_i | \sum a_i = 1, a_i \geq 0\}$ contains the 3 special cases :

a) $a_\ell = 1, \quad a_i = 0 \quad$ for $\quad i < \ell$,

b) $a_i = 1/\ell \qquad i \leq \ell$,

c) $a_i = \dfrac{\alpha^{i-1}(1-\alpha)}{(1-\alpha^{\ell})}, \quad 0 < \alpha < 1, \qquad i \leq \ell.$

Case b) gives the arithmetic average and case c) gives B_n as a truncated exponentially weighted average of the past demands (see, for example, Brown [1959]). The model in section 2 is the case a) with $\ell = 1$. In all cases given by (12), B_n is an unbiased estimate of μ, the expected demand in a period.

A key to our analysis in this section is the following

<u>Lemma</u>: For fixed $\ell \geq 1$, let the budget B_n be given by (12), and let $A_i = \sum_{j=i}^{\ell} a_j$. If we define

$$V_n = \sum_{i=1}^{\ell} A_i X_{n-i+1}, \qquad n = -1, 0, 1, \ldots, \tag{13}$$

Therefore,

$$P\left[D_{n+1} > x\right] = P\left[V_n - M_n > x\right]$$

$$\geq P\left[V_n - M_{k(n)} > x\right].$$

But V_n and $M_{k(n)}$ are independent and

$$P\left[M_{k(n)} > x\right] = \left[\bar{V}(x)\right]^{k(n)}.$$

Thus

$$P\left[D_{n+1} > x\right] \geq \int_x^{\infty}\left[1 - \left(\bar{V}(u-x)^-\right)^{k(n)}\right] dV(n). \tag{16}$$

Combining (15) and (16) gives

$$\bar{V}(x) - \int_x^{\infty} \bar{V}(u-x)^{k(n)} dV(u) \leq P\left[D_{n+1} > x\right] \leq \bar{V}\left(x + m(A,\ell)\right). \tag{17}$$

As $n \to \infty$, $\bar{V}(u)^{k(n)} \to 1$ for $0 \leq u \leq m(A,\ell)$

$\to 0$ for $m(A,\ell) < u$.

Thus

$$\int_x^{\infty} \bar{V}(u-x)^{k(n)} dV(u) \to V\left(x + m(A,\ell)\right) - V(x) = \bar{V}(x) - \bar{V}\left(x + m(A,\ell)\right).$$

When this result is used in (17), we have that

$$P\left[D_{n+1} > x\right] \to \bar{V}\left(x + m(A,\ell)\right).$$

By integration over x in (17), it is straight forward to show that

$$v - \int_0^\infty \bar{V}(u)^{k(n)+1} du \le E\left[D_{n+1}\right] \le \int_{m(A,\ell)}^\infty \bar{V}(u)\,du, \qquad (18)$$

where $v = E\left[V_n\right] = \mu \sum_{i=1}^{\ell} A_i$.

Taking the limit in (18), we see that

$$E\left[D_{n+1}\right] \longrightarrow (\mu - m) \sum_{i=1}^{\ell} A_i . \qquad (19)$$

Notice that, as in section 2, the deficits remain bounded without planned surplus resources $\left(\text{i.e., } E\left[B_n\right] = E\left[X_n\right]\right)$.

Now we look at the random variable N, and note that (10) still holds. Using the result in the proof of the lemma that $U_n = V_n - V_{n-1}$,

$$\{N > n\} \Longleftrightarrow \{V_1 > V_0, V_2 > V_0, \ldots, V_n > V_0\} .$$

Now

$$\{V_1 > V_0, V_2 > V_0, \ldots, V_n > V_0\} \Rightarrow \{V_\ell > V_0, V_{2\ell} > V_0, \ldots, V_{k(n)\ell} > V_0\},$$

where again $k(n) = \left[\frac{n}{\ell}\right]$, and for $n < \ell$ the event on the right hand side is taken to be the certain event. Then it is easy to show that

$$P\left[N > n\right] \le \frac{1}{k(n)+1} .$$

Of course this does not show that $E\left[N\right]$ is finite. By defining S_n to be the surplus resource at the end of period n, and using the same method as in section 1, $E\left[S_n\right] = 0$ for large n. Thus "busy periods" never end with probability 1, and hence $E\left[N\right]$ is in fact infinite.

We now look at some specific examples of the budget policy given in (12). First, let $a_i = 0$ for $i = 1, 2, \ldots, \ell-1$, and $a_\ell = 1$. Thus, $B_n = X_{n-\ell}$, which sets the budget in period n equal to the demand in period $n - \ell$. Then $A_i = 1$ for $i = 1, 2, \ldots, \ell$ and $\sum_{i=1}^{\ell} A_i = \ell$. Then $D(x) = F(x + m\ell)$, and $E\left[D\right] = (\mu - m)\ell$. Thus, if we considered a sequence of policies with parameter ℓ we see that the equilibrium deficits stochastically increase linearly with ℓ. We conclude that if such a budget policy is to be used, it is best to use the demand for the most recent period for which information is available. Notice that if the demand is not deterministic, then $\mu - m > 0$. One can think of the budgets and demands being less dependent as ℓ increases. Note that as $\ell \to \infty$ our result is consistent with that of the GI/G/1 queue with $\rho = 1$, namely that $E\left[D\right]$ become infinitely large.

Suppose that we interpret equation (12) to be an attempt to forecast the demand in period n from data on past demands. Having obtained a forecast, one would then allocate matching resources. The best (or BLUE) estimator of the expected demand

is obtained by setting $a_i = 1/\ell$, $i \le \ell$, in equation (12) and using ℓ large (recall our demands are stationary). But if the objective is to minimize deficits, we see from (19) that the optimal weights in (12) are $a_1 = 1$, $a_i = 0$, $i > 1$. These weights minimize both the expected deficit and the variance of the deficit. In fact, the reader can verify that the deficit with $\ell = 1$ is stochastically smaller than any other case which satisfies (12).

4. Interpretation in the Standard Queueing Model

The budget policies defined by (12) are physically realizeable when n in (12) indexes discrete time periods. In such cases, it is possible to know $X_{n-\ell}$ for some positive ℓ in time to set the budget for period n. In the case of the single channel queue where n indexes the customers in order of arrival and service, a policy of $B_n = X_{n-\ell}$ would imply that arrivals could be scheduled, and that the interarrival time between customers n and $n+1$, B_n, should be set equal to the service time of customer $n-\ell$, namely $X_{n-\ell}$, with $\ell = 1$ leading to the shortest delays in queue.

For simplicity, let $m = 0$. Then from (14) $D_{n+1} = V_n$ with probability 1 in steady state. Now, if $B_n = X_{n-\ell}$, then $D_{n+1} = X_n + X_{n-1} + \ldots + X_{n-\ell+1}$. The situation is drawn in fijure 1 for $\ell = 3$ and the queue in steady state. Note that

B_n and X_{n-3} occur simultaneously. Clearly, such a situation cannot be realized unless the service time X_{n-3} is known by the time customer n-3 starts service. When this is the case, customer n+1 is scheduled to arrive at the completion of service of n-3. Thus, there are always exactly 3 customers in the system.

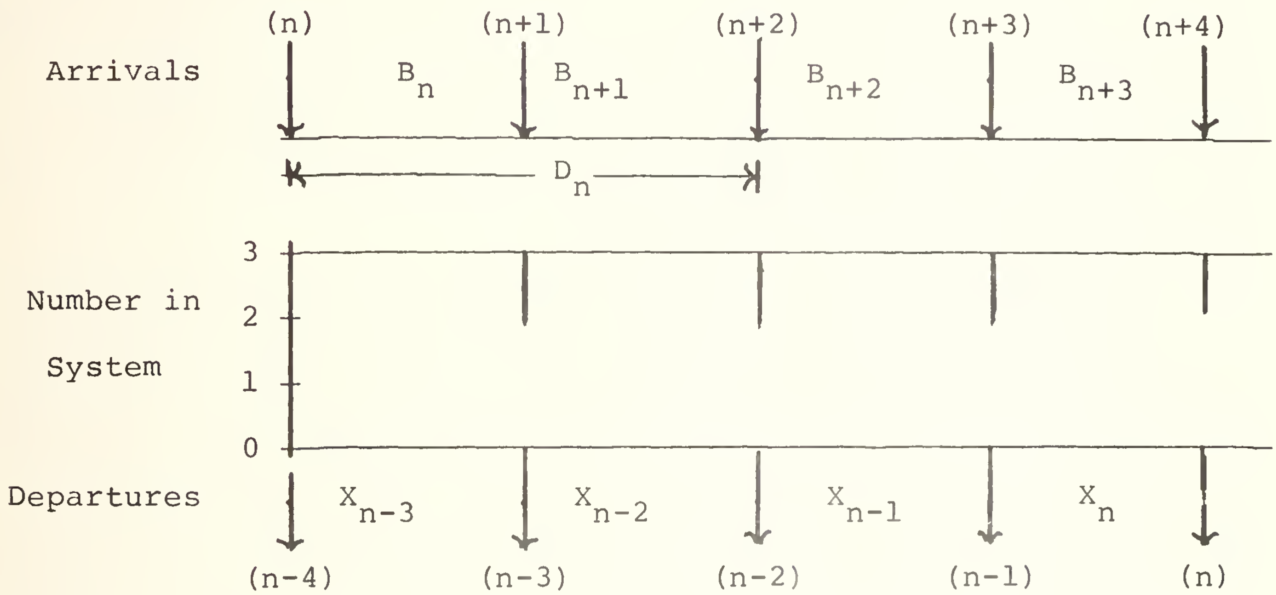

Figure 1: Steady State Realization with $\ell = 3$, $a_3 = 1$, $m = 0$.

5. Conclusions and Suggestions for Future Work.

In this paper we have identified applications of the "waiting time" random variable outside the standard applications in queueing theory and have illustrated them by examples in budgeting. To make the applications realistic, we allowed the

"service times" and "inter-arrival times" to be correlated. For a simple class of correlation relationships we determined the distribution (or bounds on the distribution) for the "waiting times" and for the "busy period" and we showed that the process with traffic intensity of unity (no excess budget) can operate 1) with small waiting "times" (deficits), 2) without planned "idle times" (surplus funds), and 3) with infinitely long expected "busy periods." We also show, where B_n is an unbiased linear estimate of μ, that it is optimal to select $a_1 = 1$, i.e., $B_n = X_{n-1}$, in order to minimize both the expected value and the variance of the "waiting times."

Because of the plethora of results for the single-channel queue with independent service and interarrival times, our results may surprise many of our readers. Certainly, most were initially non-intuitive to the authors. We see from these results some of the consequences of the assumption of independence of the sequences $\{B_n\}$ and $\{X_n\}$. In particular, we see that some forms of correlation between the two sequences enable one to schedule resource utilization very efficiently.

One can postulate more complex interrelations both between and within the sequences $\{X_n\}$ and $\{B_n\}$. For example, demands may not be iid random variables. They may be independent but growing, or they may be serially correlated. There is much evidence (see, for example, Capra [1974] and Gaver [1975]) that in governmental budgeting the appropriation of funds depends

on previous appropriations as well as demands and deficits. Although some statistical work has been carried out to demonstrate "within-sequence" correlations, the authors believe that there is much work to be done in analyzing the effects of such correlated budgeting policies on deficits and surpluses.

References

[1] Brown, R. G., Statistical Forecasting for Inventory Control, McGraw-Hill, New York, 1959.

[2] Capra, J. R., Analysis of Data Describing Congressional Responses to DOD Budget Requests, Ph.D. Thesis, Naval Postgraduate School, Monterey, June 1974.

[3] Daeschner, W. E., Models for Multi-item Inventory Systems with Constraints, Ph.D. dissertation, Naval Postgraduate School, Monterey, CA, June 1975.

[4] Gaver, D. P., DOD Budget Data Analyzed by Robust Regression Techniques, Technical Report NPS55GV75091, Naval Postgraduate School, Monterey, CA, September 1975.

[5] Kleinrock, L., Queueing Systems, Volume 1: Theory, J. Wiley and Sons, New York, 1975.

INITIAL DISTRIBUTION LIST

	No. Copies
Dean of Research Code 023 Naval Postgraduate School Monterey, CA 93940	1
Library Code 0212 Naval Postgraduate School Monterey, CA 93940	2
Library Code 55 Naval Postgraduate School Monterey, CA 93940	2
Defense Documentation Center Cameron Station Alexandria, VA 22314	12
Director Research and Development Division Code SUP 063 Naval Supply Systems Command Department of the Navy Washington, D. C. 20390	1
Dr. V. N. Bhat Dept. of Computer Science and OR Institute of Technology Southern Methodist Univ. Dallas, TX 75275	1
Dr. James Capra 7218 Delfield St. Chevy Chase, MD 20015	1
D. R. Cox Dept. of Mathematics Imperial College London, SW 7, England	1
CDR William E. Daeschner Code 94 Navy Fleet Material Support Office Mechanicsburg, PA 17055	1
Mr. Marvin Denicoff Code 437 Office of Naval Research Arlington, VA 22217	1
Dr. Saul I. Gass College of Business Management University of Maryland College Park, MD 20742	1
Dr. Richard C. Grinold Graduate School of Business Barrows Hall Univeristy of California Berkely, CA 94720	1
Professor Donald Gross School of Engineering and Applied Science The George Washington Univ. Washington, D. C. 20006	1
Carl Harris Dept. of Operations Research George Washington University Washington, D. C. 20006	1
Dr. Daniel P. Heyman Bell Telephone Labs Inc. Holmdel, NJ 07733	1
F. S. Hillier Dept. of Operations Research Stanford University Stanford, CA 94305	1
David S. P. Hopkins Academic Planning Office Stanford University Stanford, CA 94305	1
D. L. Iglehart Dept. of Operations Research Stanford University Stanford, CA 94305	

Initial Distribution List (cont'd)

	No. Copies
Dr. William S. Jewell Operations Research Center University of California Berkeley, CA 94720	1
Dr. L. Kleinrock Computer Science Dept. 3732 Boelter Hall Los Angeles, CA 90024	1
John Lehoczky Statistics Department Carnegie-Mellon Univ. Pittsburgh, PA 15213	1
Dr. J. D. C. Little Operations Research Center Room 24-215 M.I.T. Cambridge, MA 02139	1
Dr. Robert M. Oliver Operations Research Center University of California Berkley, CA 94720	1
Jim Pritchard Code NSUP 0411A Economics Analysis Branch Naval Supply Systems Command Dept. of the Navy Washington, D. C. 20390	1
Fred M. Richards 5879 Ka a Place Burke, VA 22015	1
Mr. B. B. Roseman Chief, AMC Inventory Research Office Frankford Arsenal Philadelphia, PA 19137	1
Dr. Moshe Segal Room 2D-411 Bell Telephone Labs, Inc. Holmdel, NJ 07733	1
Dr. Ronald W. Wolff Operations Research Center University of California Berkley, CA 94720	1
Dr. Thomas C. Varley (Code 434) Office of Naval Research Arlington, VA 22217	2
Dr. K. T. Wallenius Dept. of Mathematical Sciences Clemson University Clemson, SC 29631	1
Eric Wolman Bell Telephone Labs, Inc. Holmdel, NJ 07733	1
D. R. Barr, F. R. Richards R. W. Butterwoth, D. A. Schrady D. P. Gaver, B. O. Shubert G. T. Howard, M. G. Sovereign C. R. Jones, M. U. Thomas P. R. Milch, D. R. Whipple R. R. Read, P. W. Zehna Code 55 Naval Postgraduate School Monterey, CA 93940	1 ea.
F. R. Richards Code 55Rh Naval Postgraduate School Monterey, CA 93940	20 ea.
K. T. Marshall Code 55Mt Naval Postgraduate School Monterey, CA 93940	20 ea.